

First Draft of Standard on Vibration Energy Harvesting

At the 2nd Annual Energy Harvesting Workshop held on January 30 – 31, 2007, Fort Worth, TX, a committee was formed consisting members from academia, industry, and federal labs. This committee was assigned the task of compiling current practices used to characterize the vibration energy harvesting devices and come up with a metric which can allow the comparison of all prototype harvesters. This first draft of standard is just a start and will be discussed in detail at the 4th Annual Energy Harvesting Workshop to be held on January 28 – 29, 2009, at Virginia Tech (<http://www.cpe.vt.edu/ehw>). Following is the list of the committee members:

Committee Members (alphabetical order)

Bob O'Neil, Morgan Electroceramics
Brad Mitchell, Boeing
Chris Ludlow, Mide Technology Corporation
Dan Inman, Virginia Tech
Farhad Mohammadi, Advanced Cerametrics, Inc.
J. K. Huang, Ferro Solutions
Jan Kunzmann, Smart Material
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Energy harvesting: Energy recovery from freely available environmental resources. Primarily, the selection of the energy harvester as compared to other alternatives such as battery depends on two main factors: cost effectiveness and reliability. Another goal for energy harvesters has been to recharge the batteries in existing applications.

In recent years, several energy harvesting approaches have been proposed using photovoltaic, thermoelectric, electromagnetic, piezoelectric, and capacitive schemes. This first draft of standard addresses the issues related with the vibration energy harvesting which primarily utilizes electromagnetic, piezoelectric, and capacitive schemes. Most means of vibration energy harvesting (VEH) are based on a mechanically **resonant** device that is, hopefully, matched to the *vibration spectrum* of source. There are elements of various devices (inductive, piezo benders, magneto-electric) that are common, because they are all based on the forced, damped harmonic oscillator. Figure 1 shows the schematic of some of the common configurations used for piezoelectric harvesters consisting of cantilever, patches, and diaphragms.

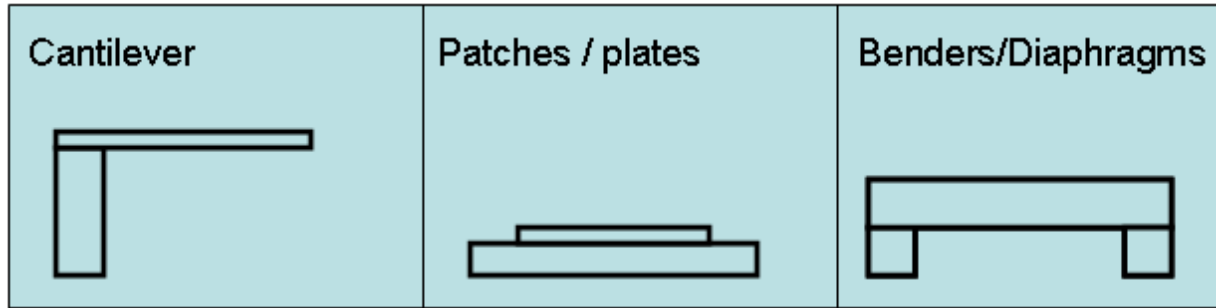


Figure 1: Common structures utilized for harvesting mechanical energy using piezoelectric transducers.

It is also possible to make **non-resonant** VEH devices. These fall into at least two categories: 1) mechanical systems that have *zero restoring force* (therefore $f_r = 0$) such as the shaker flashlights in which the freely-moving proof mass (the permanent magnet) essentially remains stationary while the flashlight case is shaken, and 2) systems designed to harvest *impact* or impulse forces.

1. *Potential vibration sources for energy harvesting*

Following is a list of vibration sources classified according to their elastic stiffness and Table I lists the sources according to the surrounding.

- stiff structures which make a movement by their own (ships, containers, mobile devices, housings of fans, escalators and elevators in public places, appliances, refrigerator, bridges, automobiles, building structures, trains)
- elastic structures which show an elastic deformation of their walls (rotor blades, wind mill blades, aircraft wings, pumps, motors, HVAC Ducts, rotorcraft)
- soft structures with very low elastic modulus and high deformation ratios (different textiles, leather, rubber membranes, piping with internal fluid flow)

Table I: Sources of energy available in the surrounding which are/can be tapped for generating electricity.

Human body	Vehicles	Structures	Industrial	Environment
Breathing, blood pressure, exhalation, body heat	Aircraft, UAV, helicopter, automobiles, trains	Bridges, roads, tunnels, farm house structures	Motors, compressors, chillers, pumps, fans	Wind
Walking, arm motion, finger motion, jogging, swimming, eating, talking	Tires, tracks, peddles, brakes, shock absorbers, turbines	Control-switch, HVAC systems, ducts, cleaners, etc.	Conveyors, cutting and dicing, vibrating mach.	Ocean currents, acoustic waves,

2. Parameters required to describe the source

- (i) The **source of vibrations** should be described to clarify the extent to which the source is diminished or degraded by harvesting some of its energy. Sources undiminished by the VEH would be earth tremors or heavy machine vibrations while the sources diminished to some extent by VEH would include small machines or an energy harvester attached to the body.
- (ii) The acceleration values for vibration source should be reported as peak to peak g level. The preferred unit for acceleration is in m/s^2 described in terms of “g” where $1g = 9.8 m/s^2$. Acceleration can be further categorized as low (less than 10 mg), mid (10 – 100 mg) and high (above 100 mg).
- (iii) The median frequency for vibration source should be reported in unit of Hz. Frequency can be categorized as low (less than 10Hz), mid (10 – 120 Hz) and high (above 120 Hz).
- (iv) For non-resonant systems, the external vibration source needs to be described (by two of the four parameters *force, displacement, velocity, acceleration*) and the resulting motion of the VEH defined. For the impact systems, the impulse, $\int F(t)dt$, must be defined as best as possible and its frequency of occurrence or duty cycle specified.

3. Theoretical models used to describe the vibration energy harvesting

(a) Williams and Yates Model¹:

The differential equation of motion describing the system in terms of the housing vibration ($y(t)=Y_o\cos\omega t$) and relative motion of mass ($z(t)$) is given as:

$$m\ddot{z}(t) + d\dot{z}(t) + kz(t) = -m\ddot{y}(t) \quad (1)$$

where m is the seismic mass, d is the damping constant and k is the spring constant. The total power dissipated in the damper under sinusoidal excitation was found to be given as:

$$P(\omega) = \frac{m\zeta Y_o^2 \left(\frac{\omega}{\omega_n}\right)^3 \omega^3}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2} \quad (2)$$

where, $\omega_n^2 = k/m$ is the system resonant frequency and $\zeta = d/2\sqrt{mk}$ is the damping ratio. If the vibration spectrum is known beforehand than the device can be tuned to operate at the resonance frequency of the system, in which case the maximum power that can be generated is given as:

¹ C. B. Williams, C. Sherwood, M. A. Harradine, P. H. Mellor, T. S. Birch, and R. B. Yates, “Development of an electromagnetic micro-generator”, IEE Proc. Circuits Devices Syst., 148 [6] 337 – 342 (2001).

$$P_{\max} = \frac{mY_o^2\omega_n^3}{4\zeta} \quad (3)$$

(b) Erturk – Inman Model:

The previous model was proposed for *electromagnetic* energy harvesters using the magnet-coil arrangement as proposed by Williams and Yates. The mechanism of *piezoelectric* transduction is relatively complicated, where the mechanical energy dissipation due to electrical power generation is not in the form of viscous damping (unlike the case in the Williams-Yates model).² The coupled distributed parameter solution of a piezoelectric energy harvester under base excitation (Figure1) was given by Erturk and Inman for unimorph³ and bimorph⁴ cantilevers (i.e., for cantilevers with one or two piezoceramic layers).

As summarized in Chapter 2 by Erturk and Inman, for the unimorph cantilever configuration, the coupled beam equation can be expressed based on the Euler-Bernoulli beam theory as

$$\begin{aligned}
& YI \frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} + \mathcal{G}v(t) \\
& \times \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = -[m + M_t \delta(x-L)] \frac{\partial^2 w_b(x,t)}{\partial t^2},
\end{aligned} \quad (4)$$

where $w_b(x,t)$ is the effective base displacement with a translational and small rotational component, $w_{rel}(x,t)$ is the transverse displacement response of the harvester beam relative to its vibrating base, $v(t)$ is the voltage response across the resistive load, YI is the bending stiffness, m is the mass per unit length, c_a is the external viscous damping term (due to air or the respective surrounding fluid), $c_s I$ is the internal strain rate (or Kelvin-Voigt) damping term, \mathcal{G} is the piezoelectric coupling term, M_t is the tip (proof) mass (if one is attached) and $\delta(x)$ is the Dirac delta function.

If the electrode pair covering the piezoceramic layer operates into a circuit with a resistive load only, the circuit equation can be obtained as

$$\frac{\varepsilon_{33}^S b L}{h_p} \frac{dv(t)}{dt} + \frac{v(t)}{R_l} = - \int_{x=0}^L e_{31} h_{pc} b \frac{\partial^3 w_{rel}(x,t)}{\partial x^2 \partial t} dx, \quad (5)$$

where ε_{33}^S is the permittivity at constant strain, b is the electrode width, L is the electrode length, h_p is the thickness of the piezoceramic layer, R_l is the load resistance, e_{31} is the piezoelectric constant and h_{pc} is the distance from the neutral axis to the center of the piezoceramic layer.

The coupled voltage response to harmonic base excitation at steady state is

² A. Erturk and D.J. Inman, Issues in mathematical modeling of piezoelectric energy harvesters, *Smart Materials and Structures*, to appear.

³ A. Erturk and D.J. Inman, A distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters, *ASME Journal of Vibration and Acoustics*, **130**, 041002.

⁴ A. Erturk and D.J. Inman, An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations, *Smart Materials and Structures*, to appear.

$$v(t) = \frac{\sum_{r=1}^{\infty} \frac{j\omega\varphi_r F_r}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega}}{\frac{1}{R_l} + j\omega C_p + \sum_{r=1}^{\infty} \frac{j\omega\varphi_r \chi_r}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega}} e^{j\omega t}, \quad (6)$$

where ω is the excitation frequency, j is the unit imaginary number, $C_p = \varepsilon_{33}^S bL/h_p$ is the internal capacitance of the piezoceramic layer, φ_r is the forward modal coupling term, χ_r is the backward modal coupling term, ω_r and ζ_r are the undamped natural frequency and the mechanical damping ratio of the r -th mode, respectively, and F_r is the modal mechanical forcing term. Thus, $|v(t)|$ is the *peak* voltage amplitude and $|v(t)|/\sqrt{2}$ is the *root mean square* voltage. Consequently, the peak power amplitude is $|v(t)|^2/R_l$ and the average power amplitude is $|v(t)|^2/2R_l$.

For modal excitations (i.e., $\omega \cong \omega_r$), the voltage expression given by Equation (6) can be simplified drastically to give the single-mode (reduced) voltage response, $\hat{v}(t)$:

$$\hat{v}(t) = \frac{j\omega R_l \varphi_r F_r e^{j\omega t}}{(1 + j\omega R_l C_p)(\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega) + j\omega R_l \varphi_r \chi_r}, \quad (7)$$

which yields the following peak power amplitude expression from $|\hat{v}(t)|^2/R_l$:

$$|\hat{P}(t)| = \frac{R_l (\omega\varphi_r F_r)^2}{\left[\omega_r^2 - \omega^2 (1 + 2\zeta_r \omega_r R_l C_p) \right]^2 + \left[2\zeta_r \omega_r \omega + \omega R_l [C_p (\omega_r^2 - \omega^2) + \varphi_r \chi_r] \right]^2}. \quad (8)$$

4. Characterization of Vibration Energy Harvester

- (i) Describe whether the VEH is uniaxial, biaxial or omnidirectional in its response.
- (ii) In order to make the comparison of various types of vibration energy harvesters, a frequency of 60 Hz and acceleration of 1g is being set as the benchmark.
- (iii) The direction of mounting for the optimum output should be specified. The type of mounting (using fasteners, glue based, permanent, nut-bolts, magnets) on the vibration source should be described.
- (iv) Following table is being given here as an example to illustrate the parameters required for VEH. This is very crucial at this stage as it provides a method for comparison of various mechanisms and designs. In this table, the volume refers to the system volume which includes all the VEH components.

Table I: Compilation of the power density and optimum operating condition for various mechanical energy harvesting devices.

Power (μW)	f (Hz)	a (ms^{-2})	Volume (mm^3)	Power density ($\mu\text{W}/\text{mm}^3$)	Acceleration ² (m^2/s^4)	Method	Mass (gm)
2	80	2	125	0.0168	4	Piezoelectric	10